Fie $f:[1,2]\to[1,2]$ o funcție continuă. Atunci:

a) $\int_1^2 \frac{1}{f(x)}\,dx + \frac{1}{2}\int_1^2 f(x)\,dx \le \frac{3}{2}$

b) Să se arate că $\int_1^2 f(x)\,dx = 2$ atunci $\int_1^2 \frac{1}{f(x)}\,dx \le \frac{1}{2}$

Demo:

a) [redacted] $1 \le f(x) \le 2 \quad \forall x \in [1,2]. \Rightarrow$

$\Rightarrow (f(x)-1)(f(x)-2) \le 0 \Rightarrow f^2(x) - 3f(x) + 2 \le 0 \Rightarrow$ 1 punct

$\Rightarrow f(x) - 3 + \frac{2}{f(x)} \le 0 \Rightarrow f(x) + \frac{2}{f(x)} \le 3 \Rightarrow$ 2 puncte

$\int_1^2 f(x)\,dx + 2\int_1^2 \frac{1}{f(x)}\,dx \le 3 \Rightarrow \frac{1}{2}\int_1^2 f(x) + \int_1^2 \frac{1}{f(x)}\,dx \le \frac{3}{2}.$ 1 punct

$1 \le f(x) \le 2 \Rightarrow \frac{1}{2} \le \frac{1}{f(x)} \le 1 \Rightarrow \frac{1}{2} \le \int_1^2 f(x)\,dx \le 1$ 1 punct

b) $\int_1^2 f(x)\,dx = 2 \overset{(a)}{\Rightarrow} 1 + \int_1^2 \frac{1}{f(x)}\,dx \le \frac{3}{2} \Rightarrow \int_1^2 \frac{1}{f(x)}\,dx \le \frac{1}{2}$ 1 punct

$\Rightarrow \int_1^2 \frac{1}{f(x)}\,dx = \frac{1}{2}.$ 1 punct

Lazărescu Dragoș